Let $\dot{\theta}$ = the angular motion of the instantaneous axis on the plane; this is the angular motion about the line (n) which is normal to the inclined plane and passes through the vertex of the cone. A plane perpendicular to the inclined plane includes this normal, the instantaneous axis (a_i), the axis of the cone, and a perpendicular (p) to the axis of the cone. The angle between n and p is α and between a_i and p is $\pi/2 + \alpha$. The components of $\dot{\theta}$ and ω about p are $\dot{\theta} \cos \alpha$ and $\omega \cos (\pi/2 + \alpha) = - \omega \sin \alpha$, but these denote the same motion; then

$$\dot{\theta} \cos \alpha = - \omega \sin \alpha \qquad \text{or} \qquad \omega = - \dot{\theta} \cot \alpha. \tag{6}$$

Now the moment of the external forces about the instantaneous axis

$$= L \cos \lambda + M \cos \mu + N \cos \nu; \tag{7}$$

also

$$= \tfrac{3}{4} mgh \sin \alpha \sin \beta \sin \theta \tag{8}$$

β being the inclination of the plane.

Substituting (4) in (3) and differentiating, we have

$$\dot{\omega}_1 = \sin \alpha(\cos \varphi \cdot \dot{\varphi}^2 + \sin \varphi \cdot \ddot{\varphi}), \quad \dot{\omega}_2 = \sin \alpha(- \sin \varphi \cdot \dot{\varphi}^2 + \cos \varphi \cdot \ddot{\varphi}), \quad \text{and} \quad \dot{\omega}_3 = \cos \alpha \cdot \ddot{\varphi}, \tag{9}$$

Now put the values given by (3), (5), (9) in (1), and we have the values of L, M, N; and using these, with (2), (7) and (8), we have the single equation of motion

$$\frac{3m}{20} (6 + \tan^2 \alpha) h^2 \sin^2 \alpha \cdot \ddot{\varphi} = \frac{3m}{4} gh \sin \alpha \sin \beta \sin \theta. \tag{10}$$

Now from (4) and (6),

$$\ddot{\theta} = - \frac{g}{h} \frac{5 \sin \beta \cos \alpha}{1 + 5 \cos^2 \alpha} \sin \theta. \tag{11}$$

If θ be so small that θ may displace $\sin \theta$, we have the required time of oscillation

$$T = 2\pi \sqrt{\frac{1 + 5 \cos^2 \alpha}{5 \sin \beta \cos \alpha} \cdot \frac{h}{g}}.$$

Also solved by F. L. WILMER, and discussed by C. H. ECKART.

2820 [1920, 134]. Proposed by C. B. HALDEMAN, Ross, Ohio.

Given one angle and the radii of the inscribed and circumscribed circles, to construct the triangle geometrically.

SOLUTION BY A. V. RICHARDSON, Bishop's College, Lennoxville, Quebec.

With the usual notation, A, R, r are given. Draw a circle of radius R, and let KB be any diameter. (The reader is requested to draw the figure). Make the angle, $BKC = A$. Draw XY parallel to BC and at a distance r from it on the same side as K. Bisect the arc BC at V, and let the circle, center V, radius VC, cut XY at I and I'.

These points will be the incenters for the two (symmetrical) solutions. Let VI meet the circle again in A. Then ABC is the required triangle. To show this, it is only necessary to prove that the straight line CI bisects the angle C. Since $VI = VC$, $\angle VIC = \angle VCI$. But $\angle VIC = (A/2) + \angle ICA$ and $\angle VCI = (A/2) + \angle BCI$. Hence $\angle ICA = \angle BCI$ and I is the intersection of two bisectors of the angles of the triangle ABC.

Also solved by L. C. MATHEWSON, H. L. OLSON, ARTHUR PELLETIER, J. B. REYNOLDS, JOSEPH ROSENBAUM, C. N. SCHMALL, and the Proposer.

2829 [1920, 226]. Proposed by E. S. PALMER, New Haven, Conn.

Given a set of arbitrary pairs of positive integers (a_p, b_p), $(p = 1, 2, \cdots, n)$: (*a*) Is it always possible to find a set of positive integers k_p, $(p = 1, 2, \cdots, n)$ such that

$$k_p a_p + k_p b_p > \sum_{r=1}^{r=n} k_r a_r, \qquad (p = 1, 2, 3, \cdots, n).$$

(*b*) If or when possible, show how to find k_p.